Lec 18: 10/28/2013

Dark Matter:

There are various lines of evidence that most of the matter in the universe is not luminous (hence "Dark Matter"). Historically, the first piece of evidence came from the rotation curves of satellites of the galaxies (and clusters). However, additional pieces of evidence have been provided in recent years from the weak gravitational lensing, large scale structure, and the CMB. In fact, we now from various lines of evidence that most of the dark matter must be non-baryonic (about half of the baryons in the universe are also "dark" as they are not luminous).

The nature of dark matter remains as one of the most prominent problems in 21st century cosmology. All of the lines of evidence for dark matter infer its existence based on gravitational interactions. Actual discovery of dark matter, which will provide the smoking gun, relies on interactions that are much stronger than gravity. Although there are many models for particle dark matter, due to lack of any discovery yet, we do not know what interactions the dark matter has. It is clear that it cannot have electromagnetic or strong interactions. In the former case, it would be luminous. In the latter case, dark matter particles would bind with protons and neutrons to form new nuclei, which would show up as anomalous isotopes of the existing elements. The very stringent experimental bounds rule out the possibility of dark matter having strong interactions.

As a result, dark matter can have interactions that are "weak" or weaker than the "weak interactions". As we saw, weak interactions freeze out at $t \sim 1$ sec ($T \sim 1$ MeV). This implies that dark matter particles decouple from the primordial plasma at a time $t_{dec} \lesssim 1$ sec. Once decouple, dark matter particles do not feel any pressure. In principle, the overdensity in dark matter could grow due to gravitational attraction at $t > t_{dec}$. However, as we will see soon, matter fluctuations do not grow significantly until t_{eq} (matter-radiation equality). However, in the interval $t_{dec} < t < t_{eq}$ dark matter particles move freely. The initial distribution of velocities is set by that of the plasma at t_{dec}. Due to random phases, the dark matter particles move in different directions. This results in dampening of the overdensities. The free streaming due to collisionless motion of particles is called "Landau damping". We expect that this effect be more important in the case that dark matter particles are in the relativistic regime at the time of decoupling. The reason being that they have larger velocities, which allows them to travel larger distances in this case. We therefore consider this case and calculate the free-streaming length λ_{FS}. It is defined as the comoving distance that dark matter particles travel between t_{dec} and t_{eq}.

The free-streaming length is given by:

$$\lambda_{FS} = \int_{t_{dec}}^{t_{eq}} \frac{v(t)\,dt}{a(t)} \qquad v(t) = \frac{\frac{p(t)}{m}}{\sqrt{1+\frac{p^2(t)}{m^2}}}$$

The integral can be performed by noticing that the physical momentum p is redshifted as $p(t) \propto a^{-1}(t)$. It will be easier to make an approximation as follows:

$$\lambda_{FS} \sim \int_{t_{dec}}^{t_{NR}} \frac{dt}{a(t)} + \int_{t_{NR}}^{t_{eq}} \frac{a_{NR}}{a^2(t)}\,dt \qquad (I)$$

Here t_{NR} denotes the time at which the dark matter particles make a transition from the relativistic to non-relativistic regime. We note that $v(t) \sim 1$ for $t_{dec} \leq t < t_{NR}$, hence the first term on the right-hand side of Eq. (I), while $v_{(t)} \propto \frac{1}{a(t)}$ for $t_{NR} \lesssim t \lesssim t_{eq}$ resulting in the second term on the right-hand side of Eq. (I).

We note that the universe is in the radiation-dominated phase for $t \lesssim t_{eq}$. This implies that $a_{(t)} \propto t^{\frac{1}{2}}$, and hence:

$$a_{(t)} = a_{NR} \left(\frac{t}{t_{NR}}\right)^{\frac{1}{2}}$$

After substituting this in the two integrals in Eq. (I), we find:

$$\lambda_{FS} \sim \frac{2\, t_{NR}}{a_{NR}} + \frac{t_{NR}}{a_{NR}} \ln\left(\frac{t_{eq}}{t_{NR}}\right)$$

We also note that $t_{NR} \propto T_{NR}^{-2}$ in the radiation-dominated phase, where T denotes the temperature of the plasma. However, temperature of dark matter particles T_{DM} is generally different from T because of decoupling of dark matter at $t \lesssim 1$ sec ($T \gtrsim 1$ MeV).

Temperature of the dark matter particles at t_{NR} can be found by equating the average kinetic energy $\simeq 3T_{DM}$ with the rest energy m_{DM} (i.e., dark matter mass). At the time t_{NR} we have:

$$T_{DM} \sim \frac{1}{3} m_{DM} \quad , \quad t_{NR} \simeq \frac{M_P}{2\left(\frac{\pi^2}{30} g_*\right)^{\frac{1}{2}} T^2}$$

Since $T \ll 1$ MeV, we have $g_* = 2 + \frac{7}{8} \times 2 \times \left(\frac{4}{11}\right)^{\frac{4}{3}} N_\nu^{eff}$, where $N_\nu^{eff} \simeq 3$ for the standard model. Putting all the pieces together, we then find:

$$t_{NR} \sim 1.2 \times 10^{7} \left(\frac{1\,\text{keV}}{m_{DM}}\right) \left(\frac{T_{DM}}{T}\right)^2 \quad \text{sec}$$

$$a_{NR} \sim 7.1 \times 10^{-7} \left(\frac{1\,\text{keV}}{m_{DM}}\right) \left(\frac{T_{DM}}{T}\right) \qquad (\text{II})$$

We note that $\frac{T_{DM}}{T}$ remains a constant at $t \gg t_{dec}$. We can also use the expression for t_{NR} and the fact that $t_{eq} \simeq 50{,}000$ yr to find:

$$\frac{t_{eq}}{t_{NR}} \sim \left[\left(\frac{10\,\text{eV}}{m_{DM}}\right)\left(\frac{T_{DM}}{T}\right)\right]^{-2} \qquad (\text{III})$$

Eqs. (I, II) combined with this expression result in:

$$\lambda_{FS} \sim 0.2 \, \text{Mpc} \left(\frac{1\,\text{keV}}{m_{DM}}\right) \left(\frac{T_{DM}}{T}\right) \left[\ln\left(\frac{t_{eq}}{t_{NR}}\right) + 2\right] \qquad (\text{IV})$$

It is seen that a smaller m_{DM} leads to a larger λ_{FS}. This is expected intuitively since a lighter particle will be in the relativistic regime for a longer period of time and travels a larger distance.

It is clear from our discussion that density fluctuations with a size $< \lambda_{FS}$ will be essentially washed out within the interval $t_{dec} < t < t_{eq}$. The smallest cosmological structures that we see today (galaxies) could not have formed if λ_{FS} had exceeded the typical size of the galaxies.